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DAMAGE ASSESSMENT OF MULTIPLE CRACKED BEAMS: NUMERICAL RESULTS AND EXPERIMENTAL VALIDATION

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A method is presented in this paper which uses the modal parameters of the lower modes for the non-destructive detection and sizing of cracks in beams. Using a finite element model of the structure to calculate the dynamic behaviour analytically, it is possible to formulate the inverse problem in optimization terms and then to utilize a solution procedure employing genetic algorithms. The damage assessment technique has been applied both to simulated and to experimental data related to cantilevered steel beams, each one with a different damage scenario; i.e., the position and depth of the cracks. It is demonstrated that this method can detect the presence of damage and can estimate both the crack positions and sizes with satisfactory precision. The problems related to the tuning of the genetic search and to the virgin state calibration of the model are also discussed.

1. INTRODUCTION

Structural components are often subjected to damage which can potentially reduce their safety; if the load-carrying capacity of the structure is not exceeded, the component does not fail and it is possible to measure and analyze its dynamic response in order to detect the damage. The concept behind Vibration Based Inspection (VBI) of structures is based on the analysis of this vibration response signal. For example, by observing the variations in the response spectra one can try to identify the element with the damage and to quantify its extent. Currently, research has been undertaken to develop VBI techniques for a wide range of applications such as beam structures [1–15], truss structures [16, 17], offshore platforms [18, 19] and rotating machinery [20, 21]. In the past decade, several algorithms have been developed which relate the changes in the spectrum to the location and size of the damage [22, 23]; while some methods perform a pattern recognition, others employ modal properties of the structure.

Many authors have addressed the damage assessment problem by trying to develop procedures based on modal methods [22, 23]. In most studies natural frequencies and mode shapes of the structure are estimated and compared to reference values related to the undamaged structure. This approach has the great advantage that modal properties are global entities and as a consequence is it possible to detect the damage across the structure.

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Nevertheless, if modal methods are used, it is usually necessary to develop a reliable mathematical model of the structure which represents the damage in terms of its geometrical properties, a task which is always difficult and occasionally impossible to perform.

The aim of this paper is to present a technique for multiple crack identification, with application both to several simulated case studies and to experimental data obtained from cantilever steel beams. Furthermore, the importance of the virgin state calibration of the model is highlighted.

2. BACKGROUND

Following pioneering work undertaken two decades ago [1, 2], many researchers have addressed the problem of determining crack position and depth of so-called non-closing fatigue cracks via dynamic characterization of the structure under investigation, concentrating in particular on applications in which a single crack is present in the structure. In this case, the method proposed by Cawley and Adams [1], another one proposed by Gudmunson [2] and the work of Liang *et al*. [3, 4] are of great importance. In particular, Gudmunson [2] evaluated the effect of a crack, notch or other geometrical imperfection on the eigenvalues through perturbation analysis. Gudmunson [2] and Liang *et al*. [3, 4] confirm the Cawley and Adams technique, showing that the ratio of two natural frequency changes is a function only of the crack position. As a consequence, the crack sizing and location tasks for a single-cracked beam are relatively straightforward in that it would be possible to address independently the tasks of quantification and localization.

If the structure is cracked in at least two positions, the problem of crack sizing and location becomes decidedly more complex. Indeed, in this case it is necessary to estimate two positions and two depths and therefore more robust and complex techniques are needed.

The double-crack assessment for beam structures has been addressed by relatively few authors. Ostachowicz and Krawczuk [6] have studied the forward problem, evaluating the changes in dynamic behaviour when the damage is known, considering a continuous model of the beam in which the cracks were modelled by introducing massless rotational springs.

By considering the transverse vibration of a cracked slender beam, Rizos *et al*. [7] obtained a system of equations for the frequencies and mode shapes in terms of crack depth and position. According to this technique, the crack may be identified by exciting the beam at a natural frequency and measuring the vibration at two amplitudes.

Surace *et al*. [8] compared the results obtained using both the continuous model and a finite element representation of the cracked beam. In a subsequent paper, Ruotolo *et al*. [9] described experimental tests carried out on cantilevered steel beams with two cracks and performed a systematic study by correlating crack locations and sizes to the corresponding changes in natural frequencies showing good agreement in the theoretical and experimental results. The conclusion was that the finite element model is more accurate for determining the variations in frequency.

Stubbs and Osegueda [10, 11] presented a method for structural damage identification that relates changes in the natural frequencies to changes in member stiffnesses with a sensitivity relation. Moreover, they demonstrated that this sensitivity method becomes difficult when the number of modes is much fewer than the number of damage parameters.

In reference [12], Hu and Liang proposed a two-step procedure to identify cracks in beam structures. They used the effective stress concept coupled with Hamilton's principle to derive a formulation equivalent to the Stubbs and Osegueda sensitivity equations. By using this formulation the elements of the structure that contain cracks could be identified, and then a spring damage model was used to quantify the location and depth of the crack in each damaged element.

The problem of a structure with multiple cracks has received, as yet, less attention. Looking at practical applications, the number of cracks actually present in a structure will usually be unknown. As a consequence the development of a generalized state-of-damage identification procedure for multiple cracks would be of considerable interest.

A typical approach for damage identification is based on optimization techniques in which the objective function to be minimized is expressed in terms of the difference between the measured and analytical characteristic parameters related to the dynamic behaviour.

Shen and Taylor [13] used a least-squares minimization and a min.–max. problem formulation in their simulation studies regarding a simply supported beam containing a crack at mid-span. In some cases poor results are obtained, probably due to the presence of multiple local minima for the objective function.

Rytter *et al*. [14] used the difference between the five lowest natural frequency ratios of the damaged and undamaged structure to locate and quantify a crack in a cantilever beam. The analysis showed that the objective function has local minima, such that it is possible to obtain a different solution to the damage assessment problem for each run of the optimization procedure starting from a different state of damage.

Davini *et al*. [15] used an objective function which measures the distance between the natural frequencies of the model and of the structure in damaged conditions, but the results exhibited high dependency on the choice of the initial point and on the minimization strategy.

In numerous previous studies, it has been observed that these functions usually exhibit local minima which would result in inaccurate identification of the damage actually present. This limitation restricts the use of classical non-linear optimization techniques and therefore favours the implementation of more recently developed global minima-seeking procedures such as genetic algorithms [24, 25]. In various applications such algorithms have demonstrated potential for the development of a rubust structural optimization procedure capable of solving even problems of considerable complexity, assuming an appropriate objective function has been selected, since the entire parameter domain can be explored and local minima avoided.

Recently, genetic algorithms have been used in order to solve many optimization problems related to the structural engineering field.

Keane [26] used genetic algorithms to optimize the structural design of a ten-bay truss structure in order to reduce the energy level of vibrations in a given frequency range.

Surace and Mares [16, 17] applied genetic search for damage detection in a reticular structure using the concept of residual force to formulate the objective function. A genetic approach for damage identification in beam structures was proposed by Friswell *et al*. [27] using an objective function with the natural frequencies and the MAC between mode shapes of the damaged structure and of the model.

Carlin and Garcia [28] determined the optimal parameters, i.e., population size, crossover and mutation probability, for structural damage detection analyzing mass–spring systems.

In a recent paper, Ruotolo and Surace [29] outlined the possibility of developing a multiple cracks identification technique using genetic algorithms, demonstrating that the actual state of damage of the structure can be estimated only if some "fundamental" functions are properly combined and a weight constraint introduced which limits the total damage. The results of this research are summarized and extended in this paper.

3. MATHEMATICAL MODEL OF THE CRACKED BEAM

To study the inverse problem (i.e., identification of the state of damage when the dynamic response changes are known) many authors have used models that simulate damage through uniform reduction in the stiffness of a relevant component. A mathematical model of this type cannot correctly define the dynamic behaviour of a cracked beam, because the reduction in natural frequencies is directly proportional to notch width, as demonstrated by Cawley and Ray [30]. Consequently, it was decided to adopt the finite element proposed by Gounaris *et al*. [31], which enables the change in natural frequencies induced by a crack to be described accurately [9].

In the majority of works aimed at diagnosing the state of damage of a beam, the structure is considered to be affected by one single crack; only very few studies assume the presence of two cracks, while the possibility of several cracks is almost invariably ignored.

Focusing attention on this last aspect, it must be emphasized that the extent and position of several cracks in a structure may be estimated only if the mathematical model does not limit the maximum number of cracks allowed. One such mathematical model can be obtained by using the finite element method, assuming that the various cracks do not interact with one another and that all components that make up the structure are potentially damaged. Subsequently, the entire structure is represented using elements with a stiffness matrix which is a function of crack size as indicated in [31]; the stiffness matrix of an element can be expressed as:

$$[K_e] = [K_e\,(r_i)] \tag{1}$$

for $r_i = a_i\,/h$, where a_i is the depth of notch and h is the beam height. Clearly,

$$r_i \in [0,\, 1[\tag{2}$$

On assembling the stiffness matrix for the entire structure, shown in Figure 1, it is possible to write:

$$[K] = [K(\tilde{R})], \tag{3}$$

where $\tilde{R}$ is a vector representing the state of damage of the structure,

$$\tilde{R} = \lfloor r_1\; r_2\; r_3 \cdots r_n \rfloor^{\mathrm{T}}, \tag{4}$$

and n indicates the number of elements that make up the structure.

As may be expected, according to the formulation of $[K_e\,(r_i)]$ given in reference [31], the following relation holds:

$$\lim_{r_i \to 0} [K_e\,(r_i)] = [K_e^u], \tag{5}$$

where $[K_e^u]$ indicates the stiffness matrix of an uncracked element of a Euler–Bernoulli beam [32]. Therefore, expression (3), which introduces a suitable value of $\tilde{R}$, applies both to a

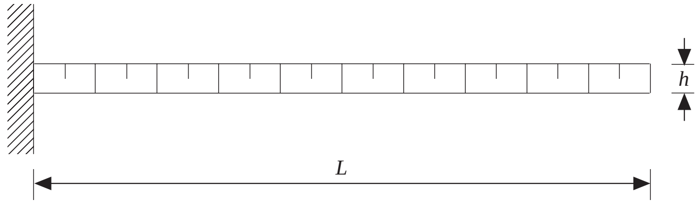

Figure 1. The multi-cracked beam.

damaged beam and to an undamaged beam, and may therefore be used to evaluate any structural damaged status.

If the damage does not affect the mass of the structure, the mass matrix may be calculated as shown in [32].

4. DEFINITION OF THE OBJECTIVE FUNCTION

4.1. GENERAL CONCEPT

In many studies, the problem of identifying structural damage reverts back to an optimization problem through the introduction of a suitable objective function. In the majority of cases this function incorporates, as numerator (or denominator), the difference between measured dynamic characteristics of the structure and the same characteristics evaluated using a mathematical model. Consequently, if the objective function is minimized (or maximized), the mathematical model exhibits the same characteristics as the actual structure, therefore reflecting its state of damage.

From previous studies on this subject [13–15], it seems usual that objective functions formulated using the dynamic behaviour of the structure examined before and after the damage have local minima, and thus any conventional hill-climbing optimization procedure would determine a wrong solution. Even if the identification problem of the damaged status is approached by optimizing an objective function through advanced procedures for determining the global minimum (or maximum), in numerical terms it emerges that not all cost functions lead to accurate estimates of the damaged status [29]. Accordingly, in this work an approach is proposed based on the formulation of the objective function by suitably combining a number of fundamental functions.

4.2. FUNDAMENTAL FUNCTIONS

In order to address the task of formulating the objective function, a series of fundamental functions have been considered which incorporate parameters related to the damaged and the undamaged states of the structure:

1. The difference between the natural frequencies of the simulated structure and the cracked one,

$$g_1(\tilde{R}) = \sum_{i=1}^{N} \left(1 - \frac{f_i^{*(m)}/f_i^{(m)}}{f_i^{*(c)}(\tilde{R})/f_i^{(c)}}\right)^2, \tag{6}$$

where N is the number of natural frequencies, the asterisk represents the cracked structure, superscript (c) the calculated and superscript (m) the measured frequencies. $\tilde{R}$ is given in equation (4).

2. The difference between the modal curvatures of the simulated and the cracked structure,

$$g_2(\tilde{R}) = \sum_{i=1}^{N} \sum_{j=1}^{M} (\phi_{i,j}''^{*(c)}(\tilde{R}) - \phi_{i,j}''^{*(m)})^2, \tag{7}$$

where M is the number of degrees of freedom of the structure and $\phi_{i,j}''$ is the jth degree of freedom of the ith modal curvature.

TABLE 1

The simulated damage scenarios

Case number	First cracked element	Depth ratio	Second cracked element	Depth ratio
1	1	0·20		
2	5	0·20		
3	3	0·20		
4	1	0·15	4	0·10
5	1	0·20	6	0·20

3. The difference between the normalized mode shapes of the simulated and the cracked structure,

$$g_3(\tilde{R}) = \sum_{i=1}^{N} \sum_{j=1}^{M} (\phi_{i,j}^{*(c)}(\tilde{R}) - \phi_{i,j}^{*(m)})^2. \tag{8}$$

It is clear that each one of the previous functions is null when the dynamic properties of the calculated and of the cracked structure are the same.

The optimization task has been performed through the use of *Genesis* 5.0, a shareware software program written by J. J. Grefenstette, in which the genetic search is coded in C language. During the initial phase of the study, it was discovered that this code gives better results if the problem is couched in terms of maximization, such that all the objective functions that have been analyzed have the fundamental functions at the denominator.

4.3. ANALYSIS OF COMBINATIONS OF FUNDAMENTAL FUNCTIONS

In order to arrive at the definition of a suitable objective function, different combinations of the fundamental functions presented above were tested simulating five different damage scenarios (see Table 1) for the cracked structure considering only the transverse vibrations. The cantilever beam under test had the following properties: elastic modulus $E = 2{\cdot}06 \times 10^{11}$ N/m^2, mass density $\rho = 7850$ kg/m^3, length $L = 0{\cdot}7$ m, cross-sectional area $0{\cdot}020 \times 0{\cdot}020$ m^2.

After some trials with the described test cases, the genetic search was set up as follows: 3000 evaluations of the cost function; a population size of 50 individuals; a crossover probability of 0·80; a mutation probability of 0·08; a crack depth ratio encoded with seven bits (128 values allowed). As suggested by De Jong [25], due to the random nature of the genetic search, for each damage scenario the simulations were run five times to evaluate the probability that the optimization procedure would converge to the same solution. In order to simulate an actual test in which usually few of the low modes are measurable, only the first modes of the structure were analyzed, such that, if not otherwise specified, the number of considered modes and natural frequencies during all the optimizations was set to three.

$f_1(\tilde{R}) = 1/g_1(\tilde{R})$. This is the most simple objective function that can be formulated when the damage assessment problem is approached through an optimization task, as only the difference between the natural frequency ratio is taken into account. It is obvious that when $f_1(\tilde{R})$ is maximized and the global maximum is reached, the damage state vector has to be a reliable estimate of the true damage state of the structure. Running the optimization procedure with the parameters described previously, the estimated state of damage differs from the real one even if the value of the objective function is very high.

This implies a small difference between the natural frequency ratios, and consequently the estimated state of damage is assumed to be correct by the optimization procedure. The results obtained are shown in Figure 2: it can be seen that, if just one crack is present (case 2) the damage is distributed near the correct location, while for multiple cracks (case 4) many elements are assumed to be damaged.

$f_2(\tilde{R}) = 1/g_2(\tilde{R})$ *and* $f_3(\tilde{R}) = 1/g_3(\tilde{R})$. The optimization procedure led to the minimization of the distance between the actual and the estimated modal curvatures $f_2(\tilde{R})$ or mode shapes $f_3(\tilde{R})$. After 3000 evaluations, both objective functions maintained a low value and consequently the optimization did not reach either a local or a global maximum. Nevertheless, it was decided not to investigate further due to the high computational time required. The damage state vector obtained after 500 generations is shown in Figure 3(a) for function $f_2(\tilde{R})$ and in Figure 3(b) for $f_3(\tilde{R})$. In both cases the estimates of damage were completely inaccurate, as the entire beam appeared to have a succession of cracks.

$f_4(\tilde{R}) = f_1(\tilde{R}) f_2(\tilde{R})$ *and* $f_5(\tilde{R}) = f_1(\tilde{R}) f_3(\tilde{R})$. In order to input more information about the system behaviour into the optimization procedure, both natural frequencies and mode shapes were considered in these objective functions. As for $f_2(\tilde{R})$ and $f_3(\tilde{R})$, the optimization led to a maximum value for the objective function which was too low to indicate that a global maximum had been reached. Nevertheless, in two of the 25 cases analyzed (five runs × five damage scenarios), the objective function reached a high value

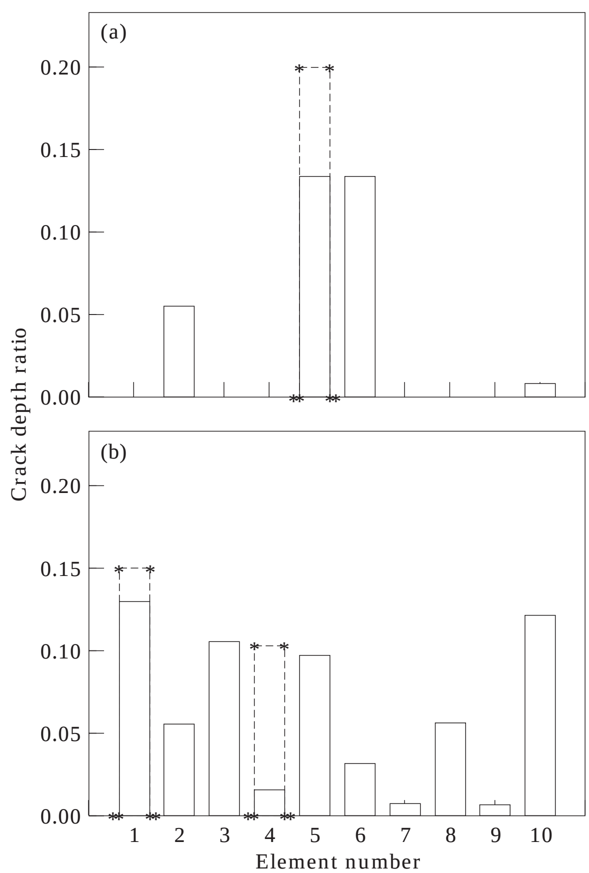

Figure 2. The results for objective function $f_1(\tilde{R})$. —, Estimated damage; –∗–, actual damage. (a) Case 2; (b) case 4.

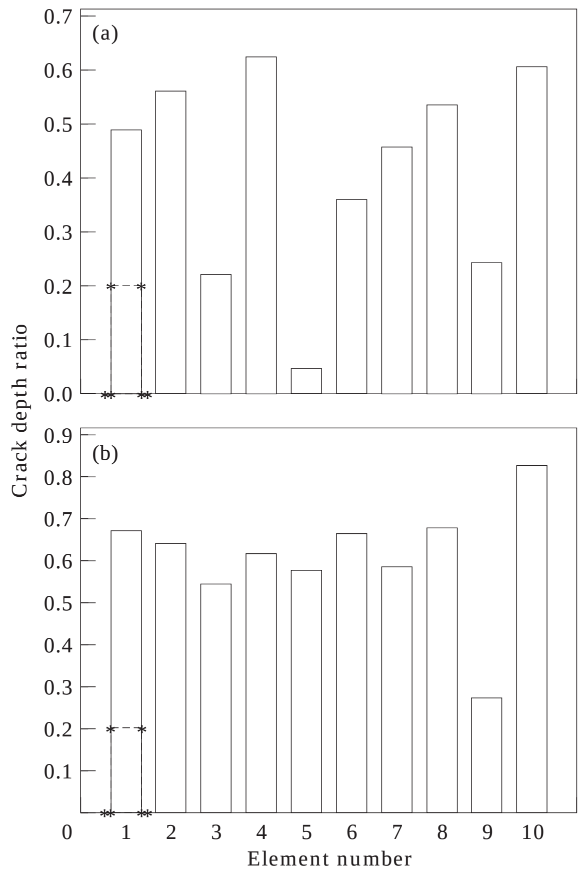

Figure 3. The results for objective functions (a) $f_2(\tilde{R})$ and (b) $f_3(\tilde{R})$, case 1. —, Estimated damage; –∗–, actual damage.

and a solution close to the correct one was obtained. These results are shown in Figure 4 for scenarios 1 and 2; it can be seen that the procedure correctly locates the damage in both cases, but more cracks are incorrectly identified.

$f_6(\tilde{R}) = [c_1 g_1(\tilde{R}) + c_3 g_3(\tilde{R})]^{-1}$. c_1 and c_3 are two coefficients properly evaluated to assure that each of the terms in the denominator have a similar value. In particular, if some simulations are run with $c_1 = 1$ and $c_3 = 10^{-7}$, the results show the presence of many cracks, such that this function gives both a localization and a quantification error even if natural frequencies and mode shapes, evaluated according to the estimate state of damage, agree well with the ‘‘experimental’’ ones.

4.4. FORMULATION OF THE OBJECTIVE FUNCTION

The formulation of this objective function has been selected as a consequence of the results described previously and considering the following observations. (a) Solving the problem of damage assessment using the optimization criterion and a mathematical model of the structure which allows the presence of a great number of cracks is very difficult. Indeed, it seems that, in a lot of cases, and even if a genetic search is used, the solution converges towards a local minimum characterized by a wrong state of damage with cracks at every site. (b) In most cases, combining the fundamental functions in a different way, i.e., adding or multiplying them, does not lead to the actual solution. (c) If the fundamental functions $g_1(\tilde{R})$, $g_2(\tilde{R})$ and $g_3(\tilde{R})$ are evaluated using the damage state vector obtained running a simulation, their value is very low; i.e., the estimated natural frequencies, mode shapes and modal curvature are very close to the measured values even if the estimated state of damage is not equally close to the real state.

Evidently, from these considerations it would be impossible further to improve the cost function by introducing terms related to the dynamic behaviour of the structure. At the same time it seems necessary to introduce a weighting term that promotes the determination of damage at fewer sites; i.e., more concentrated rather than being distributed across the beam.

Due to the previous considerations, the following objective function has been formulated:

$$F(\tilde{R}) = [v(\tilde{R}) + c_1 g_1(\tilde{R}) + c_2 g_2(\tilde{R}) + c_3 g_3(\tilde{R})]^{-1}, \tag{9}$$

where

$$v(\tilde{R}) = \sum_{i=1}^{n} r_i \tag{10}$$

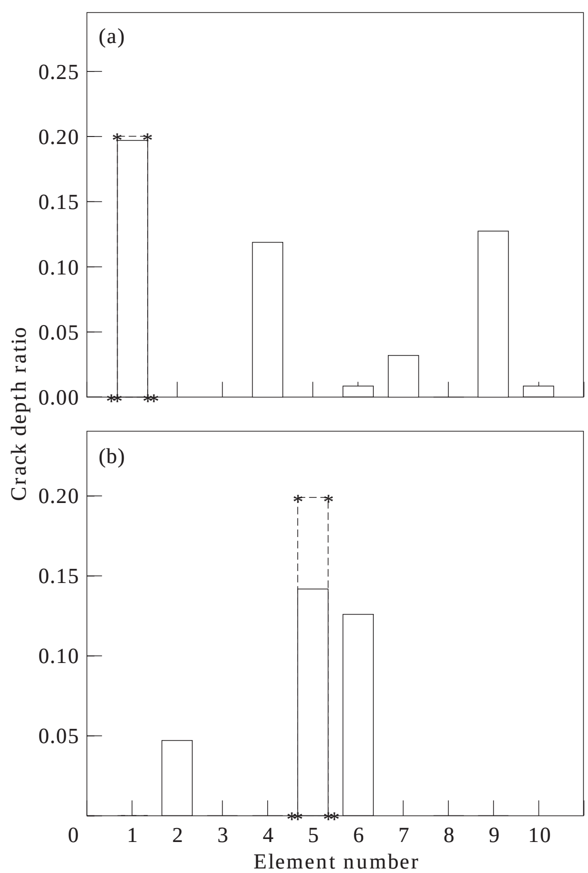

Figure 4. The results for objective function $f_4(\tilde{R})$. —, Estimated damage; – ∗–, actual damage. (a) Case 1; (b) case 2.

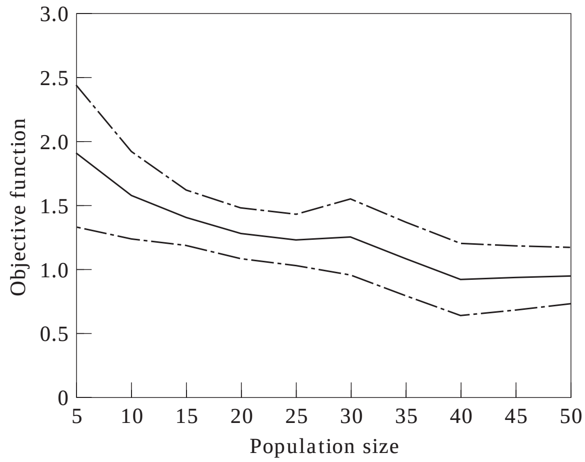

Figure 5. The mean value (—) and the mean $\pm$ the r.m.s. value (– · –) of the objective function versus population size.

denotes the global damage of the structure as the sum of damage affecting n elements making up the mathematical model; $g_1(\tilde{R})$, $g_2(\tilde{R})$ and $g_3(\tilde{R})$ are expressed as equations in (6)–(8).

The role of the function $v(\tilde{R})$ is of fundamental importance for an accurate assessment of the damage status. Indeed, in the previous section it has been highlighted that when just a combination of $g_1(\tilde{R})$, $g_2(\tilde{R})$ and $g_3(\tilde{R})$ is used as an objective function, the optimization techniques applied to multi-damaged structures tend to introduce cracks in each element in spite of the fact that the objective functions $g_1(\tilde{R})$, $g_2(\tilde{R})$ and $g_3(\tilde{R})$ are virtually zero [27, 29]. Consequently, even if the model dynamic characteristics are very close to the measured values, the damaged status cannot be equally close.

Introducing function $v(\tilde{R})$ to the denominator of the objective function, $F(\tilde{R})$, tends to reduce the global damage of the structure, so much so that maximizing $F(\tilde{R})$ implies minimizing $v(\tilde{R})$. As a consequence, if the structure under test is actually damaged, the function $v(\tilde{R})$ decreases during the optimization process until the actual state of damage is obtained, such that this constraint enables automatic localization of the cracks. Furthermore, by introducing the constraint term $v(\tilde{R})$ the objective function tends asymptotically towards the value $1/v(\tilde{R})$. Therefore, if vector $\tilde{R}$ represents the solution of the inverse problem, $F(\tilde{R})$ takes a finite value (if the structure is damaged) equal to the reciprocal of the global damage of the structure.

The expression (9) introduces a very general formulation for the cost function and enables any combination of fundamental functions $g_i(\tilde{R})$ to be used. The numerical investigation, presented in the next sections, was carried out using just natural frequencies and modal curvatures (setting $c_3 = 0$), while the experimental results were obtained analyzing just the natural frequencies (setting $c_2 = c_3 = 0$).

4.5. APPLICATION OF THE OBJECTIVE FUNCTION

The procedure for identifying damage described in this work was applied to the five simulation cases shown in Table 1. The first runs of the optimization procedure using $F(\tilde{R})$ demonstrate the capability of this objective function to give good estimates of the state of damage. As a consequence, in order to apply this procedure to experimental data, a systematic numerical investigation was performed to tune the genetic algorithm; i.e., to determine the optimum values for the following parameters: population size, crossover

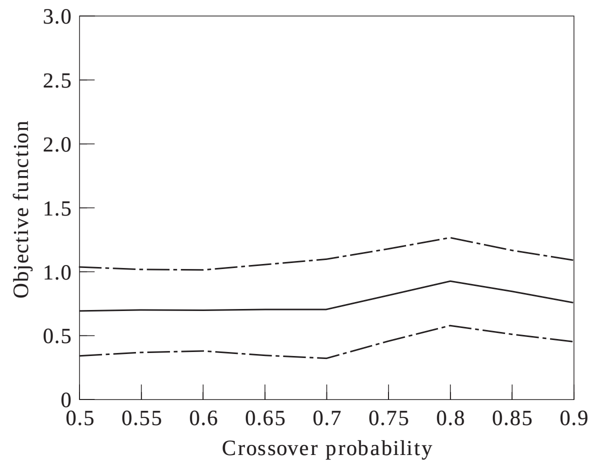

Figure 6. The mean value (—) and the mean $\pm$ the r.m.s. value (– · –) of the objective function versus crossover probability.

probability and mutation probability. These were selected through a series of optimizing operations concerning a cantilever steel beam having the same characteristics as previously mentioned, discretized using 15 elements and cracked to a depth of 20% at the element close to the clamped end. The following values were adopted for parameter tuning: population size, 50 individuals; mutation probability, 0·08; crossover probability, 0·80. The objective function $F(\tilde{R})$ had been set at $c_1 = 10\,000$, $c_2 = 100$ and $c_3 = 0$. The standard setting was subsequently modified by altering one parameter at a time and running the calculation program 20 times in order to compensate for random fluctuations reflecting the casual nature of genetic evolution. The objective function was evaluated 5000 times for each optimization process. The mean value and the standard deviation of the function were subsequently calculated at the end of optimization. The results are shown in Figures 5–7, for population size, crossover probability and mutation probability respectively. These diagrams highlight the fact that crossover probability does not have any undue effect on the maximum value of the objective function obtained during

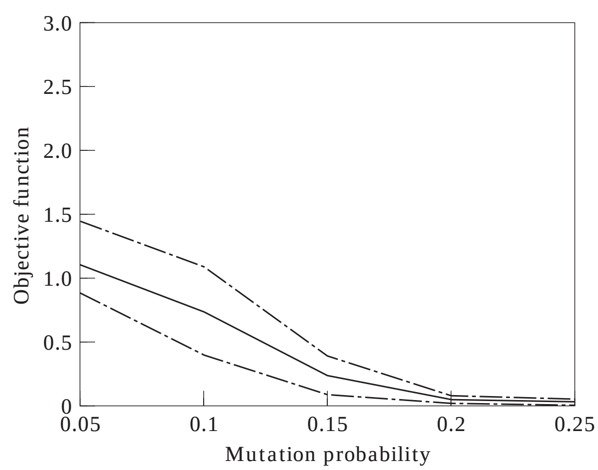

Figure 7. The mean value (—) and the mean $\pm$ the r.m.s. value (– · –) of the objective function versus mutation probability.

optimization, whereas a more satisfactory effect is obtained by altering population size and/or mutation probability.

After tuning the optimization technique, the following values were selected: a population size of ten individuals, a mutation probability of 0·05 and a crossover probability of 0·80, individuals of all populations having been randomly initialized.

Figures 8(a)–8(j) show the results of the application of $F(\tilde{R})$ to all of the analyzed damage scenarios for a beam discretized with ten elements when it is set at $c_1 = 10\,000$, $c_2 = 100$ and $c_3 = 0$, and 3000 evaluations of the objective function are performed. In all the cases, except the second, only three natural frequencies and mode shapes are used, while in the second case the actual result is obtained using at least four natural

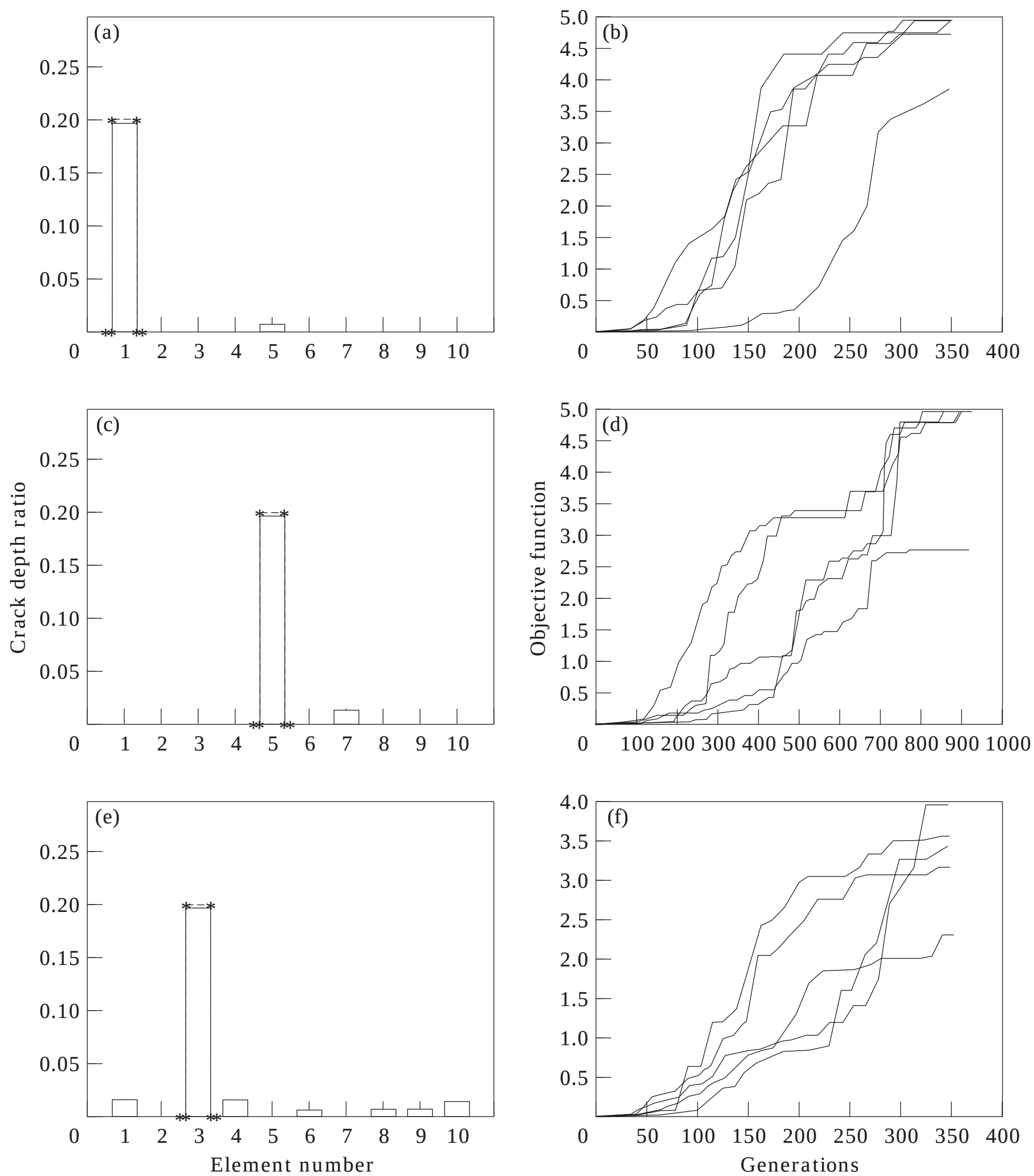

Fig. 8(a)–(f)

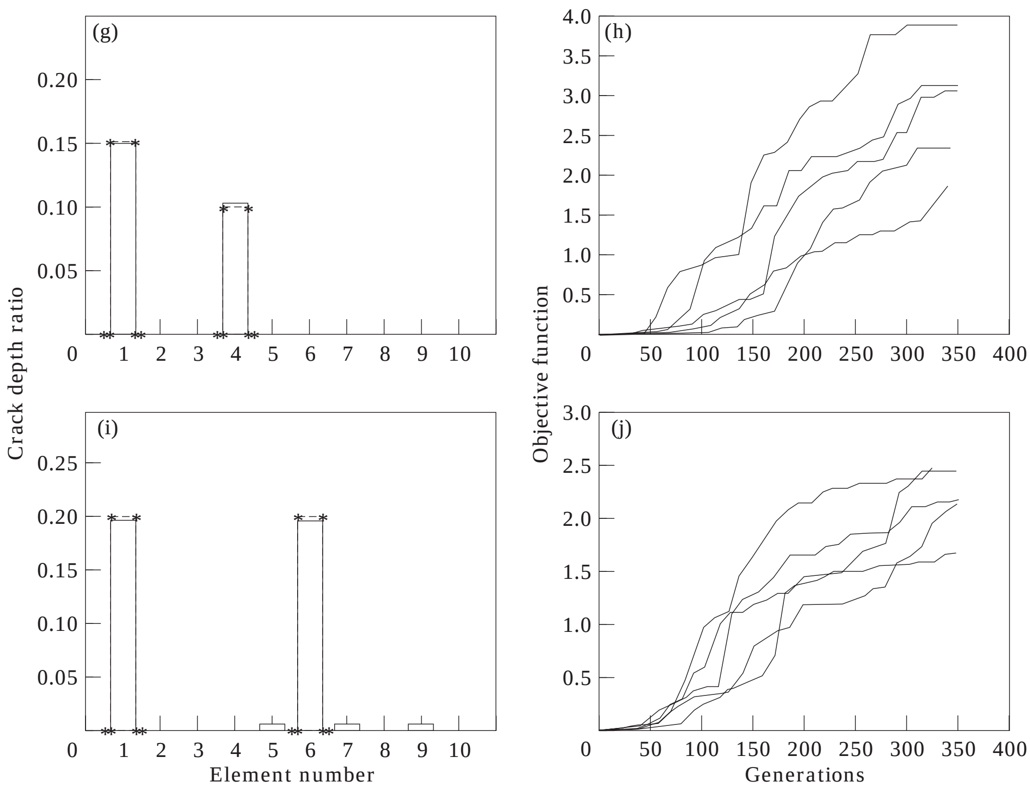

Figure 8. The results for objective function $F(\tilde{R})$. —, Estimated damage; –*–, actual damage. (a, b) Case 1; (c, d) case 2; (e, f) case 3; (g, h) case 4; (i, j) case 5.

frequencies and modes. The diagrams demonstrate that the proposed technique determines the number of cracks quite automatically, localizing and quantifying them correctly by introducing non-existing cracks in the actual structure to a maximum depth ratio of 3%.

All of the results presented are related to cracks that are effectively located in the centre of the corresponding element. In order to check the validity of the proposed method when the cracks are not in the middle of the elements, numerical simulations have been performed as follows. The "experimental" data have been generated by using a cantilever beam discretized with eight elements of equal length and with a crack in the element nearest to the clamped end. In order to identify the damage, a finite element model of the beam subdivided into 16 elements has been used, such that in this case the crack falls at the junction between the first two elements. By running the genetic search five times as previously described, the technique located two cracks, one in the first element and the other in the second, each one with half of the real crack depth.

5. EXPERIMENTAL VALIDATION

5.1. EXPERIMENTAL SET-UP

The above-described technique for identifying structural damage was validated through the analysis of data from experimental tests conducted on C30 steel beams of rectangular

TABLE 2

The experimental cantilever beams

Beam number	First crack position (m)	Depth ratio	Second crack position (m)	Depth ratio
C1	0·254	0·20	0·545	0·20
C2	0·254	0·20	0·545	0·30
C3	0·254	0·30	0·545	0·20

cross-section, $0{\cdot}02 \times 0{\cdot}02\,m^2$, 0·8 m long and clamped at one end. In order to consider different cases of damage, three beams were set up, each with two cracks with positions and depths as indicated in Table 2. By discretizing the beams using 11 elements of equal length, crack positions indicated in Table 2 lie on the centerline of elements 4 and 8. The dynamic characteristics of the beam, i.e., the natural frequencies, damping ratios and mode shapes, were estimated before and after damaging to permit accurate initial calibration of the mathematical model of the structure under test.

Cracks were obtained by wire erosion with a 0·10 mm diameter wire to produce notches 0·13 mm wide. Broadband random noise approximating to a white noise process was employed to excite the beams at the free end. Measuring the transverse acceleration of the beam at several points (11 in this case) and acquiring the force acting at its free end, it is possible to evaluate all the frequency response functions (inertances) between these positions which lead to the estimation of the bending natural frequencies. To reduce the effect of noise, several FRF's, usually 50, are averaged together to obtain a single FRF. The *least squares complex exponential* technique was applied in order to extract all the system poles and the *least squares frequency domain* procedure was used to estimate the system mode shapes [33]. The complete scheme of the measuring set-up is shown in Figure 9.

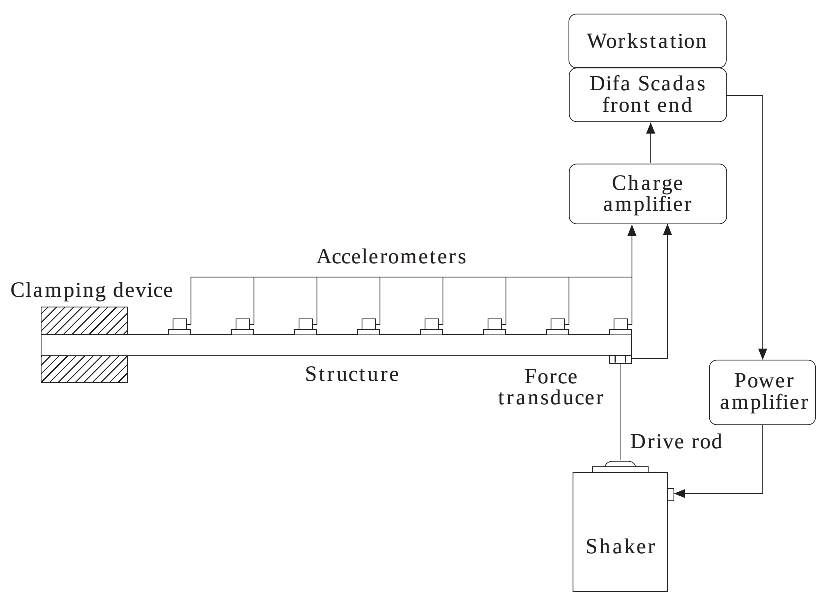

Figure 9. The test set-up.

TABLE 3

Measured natural frequencies for undamaged and damaged beams

Beam number	f_1 (Hz)	r.m.s. (%)	f_2 (Hz)	r.m.s. (%)	f_3 (Hz)	r.m.s. (%)	f_4 (Hz)	r.m.s. (%)
C1								
Undamaged	24·248	0·21	152·026	0·08	424·457	0·09	823·160	0·07
Damaged	24·066	0·18	150·612	0·16	416·579	0·12	820·399	0·21
C2								
Undamaged	24·175	0·41	152·103	0·12	424·455	0·07	824·209	0·13
Damaged	24·044	0·14	149·268	0·14	409·287	0·12	818·150	0·08
C3								
Undamaged	24·145	0·40	151·873	0·12	424·328	0·10	823·749	0·12
Damaged	23·892	0·14	150·260	0·12	411·265	0·09	819·811	0·12

The wire erosion process used to produce the cracks demands that the beams are removed from the clamping device; obviously, even if the latter is locked very accurately, the dynamic characteristics of the structure may alter as a consequence of first removing and then resetting the beam in the device. Consequently, it was decided to run ten vibration tests for each beam before and after crack machining. Thus, it is reasonable to assume that the inaccuracy introduced by the clamping device installation is mediated and, therefore, relatively insignificant.

The mean values and relative standard deviations of the first four bending natural frequencies of each beam are indicated in Table 3. In the majority of cases, the relative standard deviation of estimated values is rather low and always below approximately 0·0040. Considering that this value is decidedly low so as to cause a clear separation of variability fields of estimates for uncracked and cracked beams, it is possible to assume that the mean value of estimated natural frequencies is sufficiently accurate to warrant continuation with the technique for identifying structural damage.

5.2. VIRGIN STATE CALIBRATION

The mathematical model of the undamaged beam must be calibrated based on experimental data in order to describe the dynamic behaviour of the actual structure as accurately as possible. Obviously, any errors during measurement and assessment of dynamic characteristics or during subsequent calibration of the mathematical model may lead to an error in judgement of the damaged status.

Calibration of the mathematical model must consist mainly of two steps. Firstly, it is necessary to identify the physical parameters which more significantly influence the dynamic behaviour of the structure. Secondly, it is important to select a mathematical procedure permitting an accurate assessment of the value of these parameters when the mathematical model describes the behaviour of the actual structure.

The first step in determining the physical parameters must take into account that bending natural frequencies of a beam are proportional to flexural stiffness. Moreover, the structure to be modelled is a cantilever beam clamped by a device which does not perform in the same way as an ideal constraint; therefore, a second parameter to be considered is the stiffness of the clamping device. Finally, 11 equispaced accelerometers were placed on each beam for vibration testing; these sensors, with a little mass, can be disregarded when considering low frequency dynamics, whereas they may become significant when considering dynamic characteristics at higher frequencies. A schematic representation of the mathematical model is illustrated in Figure 10.

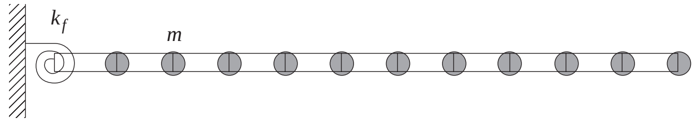

Figure 10. A schematic model for virgin state calibration.

One of the more common calibration techniques is that of minimizing the square of the difference between measured and calculated characteristics. To give the same importance to the various natural frequencies the following expression may be introduced:

$$F_c(k_f, m, EI) = \sum_{i=1}^{N} \left(1 - \frac{f_i^{(c)}(k_f, m, EI)}{f_i^{(m)}}\right)^2, \tag{11}$$

where k_f is the constraint spring stiffness, m is the mass of each accelerometer and EI is the bending stiffness. Based on definition (11), the function $F_c(k_f, m, EI)$ will exhibit a theoretically zero global minimum if the mathematical model represents an optimum actual structure.

As the function (11) is non-linear and may exhibit local minima, it was decided to proceed with optimization, using the maximizing function $1/F_c(k_f, m, EI)$, through genetic algorithms. To use this technique it was necessary to set an interval of validity for the three parameters considered. In particular, the following were selected: $EI \in [\frac{4}{3}, \frac{20}{3}] \times 10^3$ N m^2; $k_f \in [1, 5000] \times 10^3$ N m rad^{-1}; and $m \in [0, 50] \times 10^{-3}$ kg; encoding each parameter with 13 bits (8192 values are allowed within the associated interval).

In Table 4 optimum values of the parameters k_f, EI and m for the three beams considered are given, whereas in Figure 11 the maximum value of the objective function during calibration of the model for beam C2 is shown. In Tables 5 and 6 a comparison of the first four measured natural frequencies of beams C1, C2 and C3 with those obtained with a calibrated mathematical model is shown.

5.3. EXPERIMENTAL RESULTS

For the purpose of validating the technique for identifying the extent of damage proposed in Section 4 of this paper, data obtained from vibration testing were used to localize and quantify the cracks affecting beams C1, C2 and C3. On the basis of numerical results obtained during development of the identification technique, it was decided to apply the optimization procedure based on genetic algorithms by assuming the following parameters: 5000 evaluations of the objective function; a population size of ten individuals; a crossover probability of 0·80; a mutation probability of 0·05; and a crack depth ratio encoded with 13 bits (8192 values allowed). It was also decided to increase the number of objective function evaluations both to consider the experimental nature of the data and

TABLE 4

Design parameters after the virgin state calibration

Design parameters	Beam number C1	C2	C3
EI (N m^2)	2902·533	2803·560	2797·707
k_f (N m rad^{-1})	$9{\cdot}95365 \times 10^5$	$9{\cdot}91282 \times 10^5$	$9{\cdot}35360 \times 10^5$
m (kg)	$36{\cdot}67 \times 10^{-3}$	$29{\cdot}21 \times 10^{-3}$	$28{\cdot}98 \times 10^{-3}$

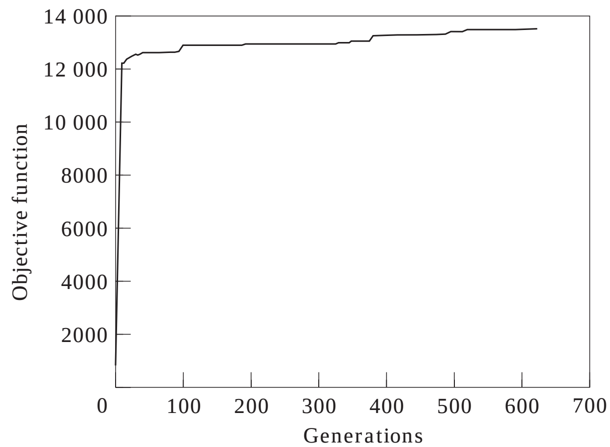

Figure 11. Virgin state calibration for beam C2.

to obtain greater accuracy. By performing a systematic analysis of the measured mode shapes before and after damage, the conclusion was reached that these were not sufficiently accurate to be considered for crack identification. Therefore, it was decided to use $c_1 = 10\,000$ and to set $c_2 = c_3 = 0$, so as to be able to consider only the natural frequencies of the beam and not to use the mode shapes or the curvatures (the latter are difficult to determine accurately from an experimental approach) in the damage identification procedure.

The optimization program was run five times for each experimental case; in Figures 12(a)–12(f) both the associated trends of the objective function for each run are shown and, considering that the five estimates of the damaged status are very similar, a comparison is indicated between actual damaged status and that obtained with the proposed technique. In all cases the identification technique automatically indicates the exact number of cracks affecting the structure and, for beams C1 and C3, localization is exact; as regards beam C2, the first crack is localized in the wrong element although this

TABLE 5

Comparison between the first and second measured and calculated natural frequencies (Hz) *for the updated mathematical model*

Beam number	$f_1^{(m)}$	$f_1^{(c)}$	Error (%)	$f_2^{(m)}$	$f_2^{(c)}$	Error (%)
C1	24·248	24·184	−0·26	152·026	151·429	−0·39
C2	24·175	24·178	0·01	152·103	151·390	−0·47
C3	24·145	24·148	0·01	151·873	151·226	−0·42

TABLE 6

Comparison between the third and fourth measured and calculated natural frequencies (Hz) *for the updated mathematical model*

Beam number	$f_3^{(m)}$	$f_3^{(c)}$	Error (%)	$f_4^{(m)}$	$f_4^{(c)}$	Error (%)
C1	424·457	423·728	−0·17	823·160	829·978	0·83
C2	424·455	423·622	−0·20	824·209	829·772	0·67
C3	424·328	423·212	−0·26	823·749	829·076	0·65

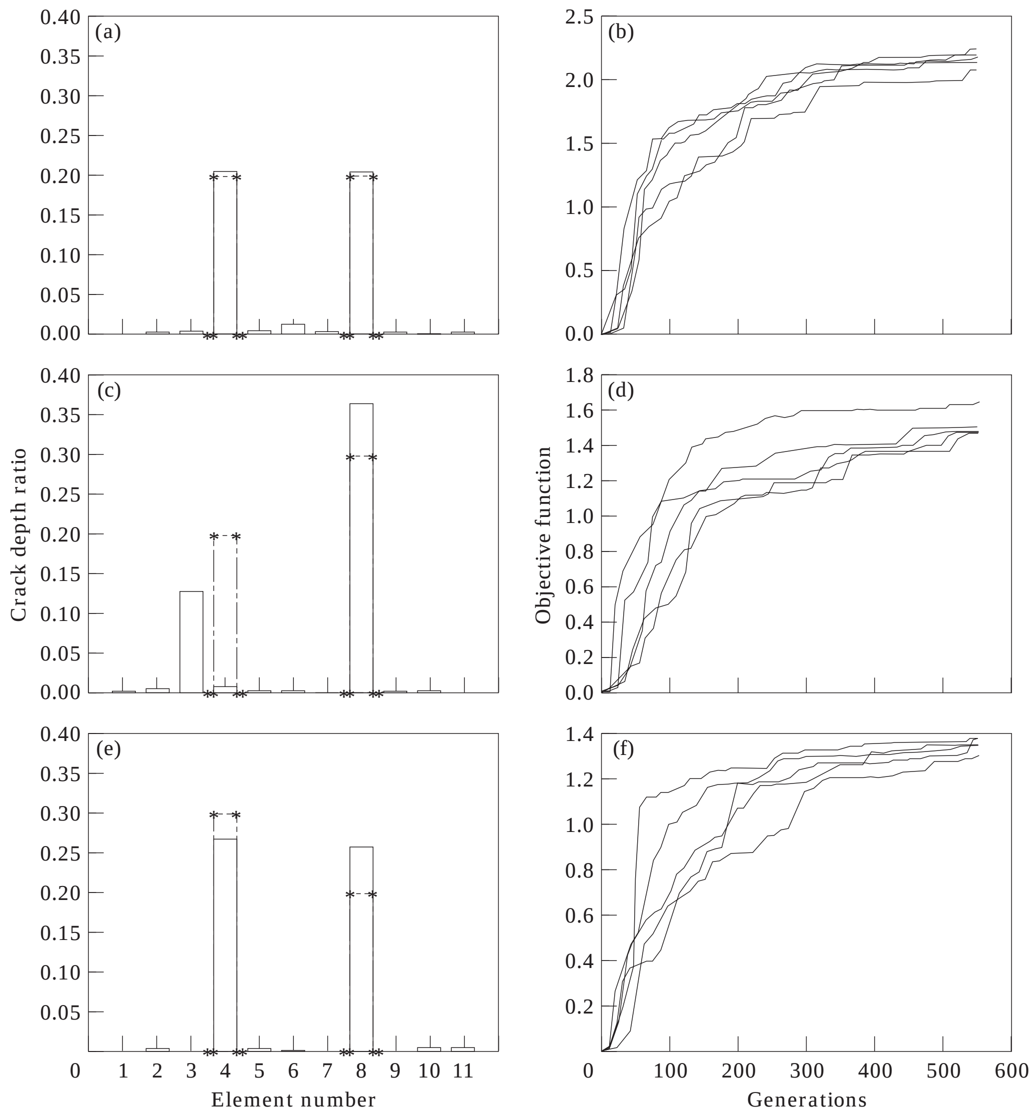

Figure 12. The results for objective function $F(\tilde{R})$ applied to experimental data. —, Estimated damage; –∗–, actual damage. (a, b) Beam C1; (c, d) beam C2; (e, f) beam C3.

is adjacent to the element actually damaged. The estimated extent of the crack is satisfactory in all of the cases.

6. DISCUSSION

In order to compare the method to other techniques capable of dealing with multiple-cracked structures, the experimental results have been analyzed by using a two-step procedure similar to that one proposed by Hu and Liang [12]. Firstly, a sensitivity approach [10] may be used to locate the damaged members; secondly, a massless rotational spring model [12] can be utilized to estimate the depth of the crack in each damaged element.

6.1. LOCATION

Assuming that a given structure, discretized into n members, undergoes a structural modification, Stubbs and Osegueda [10] demonstrated that the variation of the eigenvalue v_i is in relation to the reduction in member stiffness (proportional to α_k) according to the following relation:

$$\frac{\delta v_i}{v_i} = \sum_{k=1}^{n} S_{i,k}\, \alpha_k, \tag{12}$$

where $S_{i,k}$ gives the eigenvalue sensitivity with respect to stiffness modifications. Relation (12) can be rewritten as

$$\tilde{\mathbf{Z}} = [\mathbf{S}]\tilde{\boldsymbol{\alpha}},$$

such that if the vector of eigenvalue variation $\tilde{\mathbf{Z}}$ is evaluated experimentally, the damaged members can be located by calculating the vector $\tilde{\boldsymbol{\alpha}}$. Usually, the number of evaluated natural frequencies is lower than the number of discretizing elements. Hence $\tilde{\boldsymbol{\alpha}}$ may be calculated as follows:

$$\tilde{\boldsymbol{\alpha}} = [\mathbf{S}]^{\mathrm{T}}([\mathbf{S}]\,[\mathbf{S}]^{\mathrm{T}})^{-1}\tilde{\mathbf{Z}}. \tag{13}$$

According to reference [10], the sensitivity matrix can be evaluated for a uniform beam through the relation

$$S_{i,k} = \frac{\int_{x_i}^{x_{i+1}} \phi''_k\,(x)^2\,\mathrm{d}x}{\int_0^{\mathrm{L}} \phi''_k\,(x)^2\,\mathrm{d}x}, \tag{14}$$

where $\phi''_k\,(x)$ is the second derivate of the kth mode shape of the undamaged beam.

6.2. QUANTIFICATION

When the cracks have been located, it is possible to use a massless rotational model in order to relate the eigenvalue variation to a given function of the depth of each crack:

$$\delta v_i = -\sum_{k=1}^{n_c} \frac{5{\cdot}346h\,EI}{L^4}\,\phi''_i\left(\frac{x_k}{L}\right)^2 \gamma(r_k), \tag{15}$$

in which L is the length of the beam, n_c is the number of detected cracks, x_k is the location of the kth crack and, according to Rizos *et al*. [7],

$$\gamma(r) = 1{\cdot}8624r^2 - 3{\cdot}95r^3 + 16{\cdot}375r^4 - 37{\cdot}226r^5 + 76{\cdot}81r^6 - 126{\cdot}9r^7 + 172r^8 - 143{\cdot}97r^9 + 66{\cdot}56r^{10}. \tag{16}$$

Equation (15) can be rewritten in matrix notation as follows:

$$\tilde{\mathbf{V}} = [\mathbf{B}]\tilde{\mathbf{A}}_c,$$

where

$$V_i = \delta v_i, \qquad B_{i,k} = -\frac{5{\cdot}346h\,EI}{L^4}\,\phi''_i\left(\frac{x_k}{L}\right)^2, \qquad A_{ck} = \gamma(r_k),$$

TABLE 7

Estimated and actual size of the cracks for beams C1, C2 and C3

Beam number	a_1/h Actual	a_1/h G.A.	a_1/h [12]	a_2/h Actual	a_2/h G.A.	a_2/h [12]
C1	0·20	0·21	0·23	0·20	0·21	0·21
C2	0·20	0·13	0·18	0·30	0·37	0·33
C3	0·30	0·26	0·27	0·20	0·26	0·25

such that it is possible to evaluate $\tilde{\mathbf{A}}_c$,

$$\tilde{\mathbf{A}}_c = ([\mathbf{B}]^T[\mathbf{B}])^{-1}[\mathbf{B}]^T\tilde{\mathbf{V}}, \tag{17}$$

and, finally, it is possible to estimate the depth of the crack by solving

$$\gamma(r_k) - A_{ck} = 0.$$

6.3. ANALYSIS OF THE EXPERIMENTAL DATA

The two-step procedure proposed in reference [12] has been applied to locate and quantify the cracks present in the beams C1, C2 and C3. The application of the sensitivity technique to all the experimental cases provides the correct location of the cracks for each beam; i.e., the fourth and eighth members of the model. By using the middle point of the damaged elements as crack location, it is possible to proceed with the quantification task. In Table 7, the estimates of r_k for each beam obtained by using both the two-step procedure and the genetic search are shown, together with the actual depth ratio of the cracks. It appears that the maximum quantification error is equal to 5% of the height of the beam for the two-step approach and 7% for the genetic search. As a consequence, both the techniques provide satisfactory results. However, it should be considered that the genetic search is straightforward to apply, as it gives both position and depth of the cracks automatically, while the two-step procedure divides the damage identification into two, more simple, tasks. Moreover, the sensitivity approach has some problems in locating the damage, especially when more than one fault is present and the structure is discretized into a lot of members [22]. When applied to this case, the two-step method tends to indicate that the damage is spread over more members than those actually damaged and, as a result, it could be difficult to understand whether or not a member is effectively cracked.

7. CONCLUSIONS

The structural damage identification technique presented in this paper can address the following tasks: (1) damage detection; (2) damage location; and (3) damage sizing. The technique was initially applied to data obtained through finite element modelling, which permitted calibration of the coefficients for the cost function selected and of the parameters appearing in the optimization procedure through genetic algorithms. Subsequently, a number of experimental tests were run in controlled conditions on beams with two cracks clamped at one end for the purpose of validating experimentally the technique developed for structural damage identification. Beam cracking was produced in the form of notches of known position and depth and measurements were made of the dynamic characteristics of the specimens in the undamaged and damaged condition respectively. The mathematical model of the beam was calibrated through maximization of a specific cost function including measured data on integral structure. The calibrated model was subsequently used for identification of the damaged status; results obtained indicate that the identification technique permits assessment of the number of cracks induced on the beam, of their position and depth with satisfactory accuracy.

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